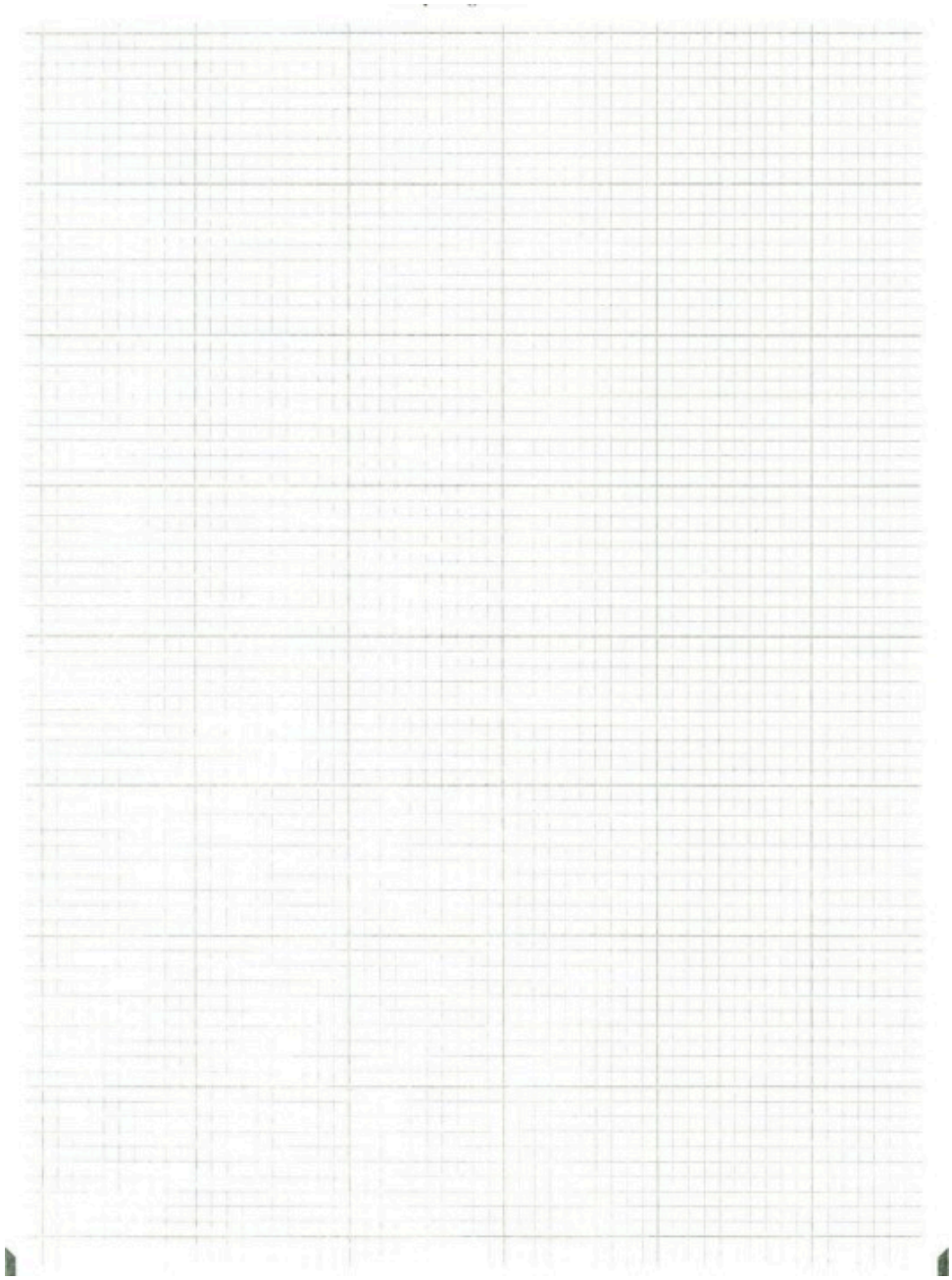
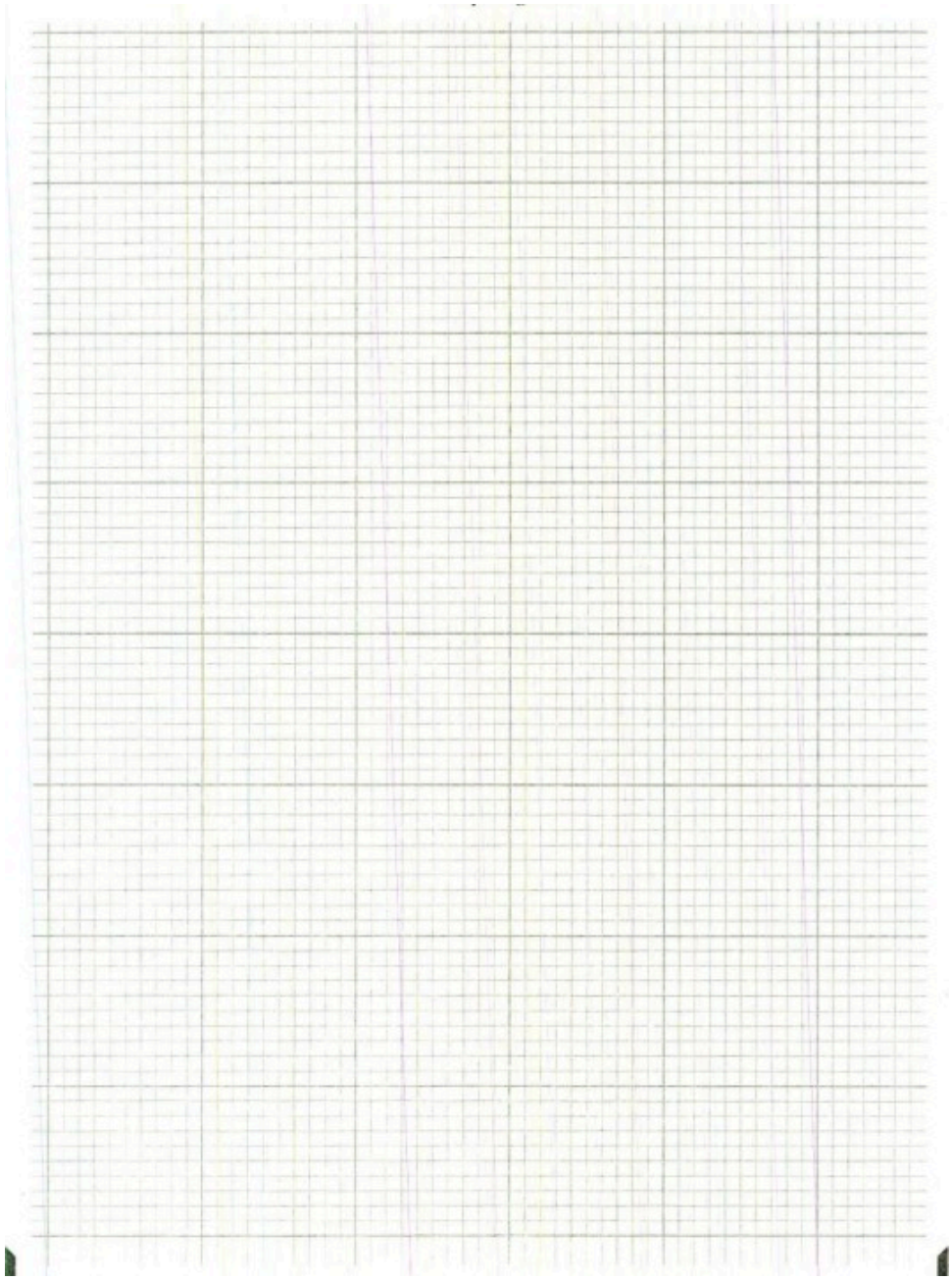
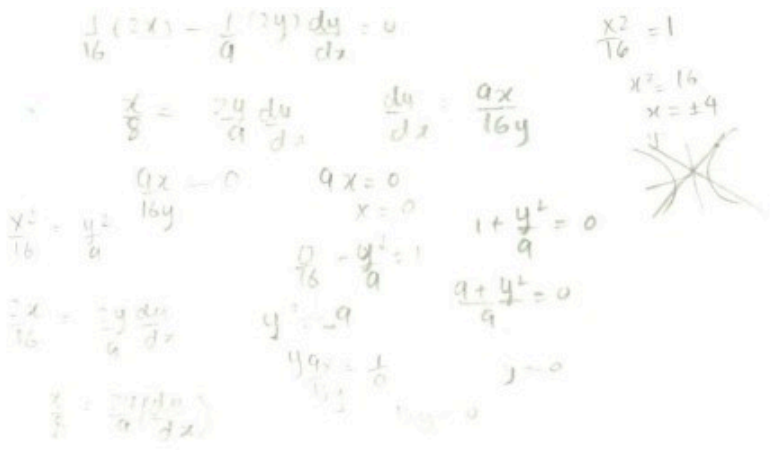
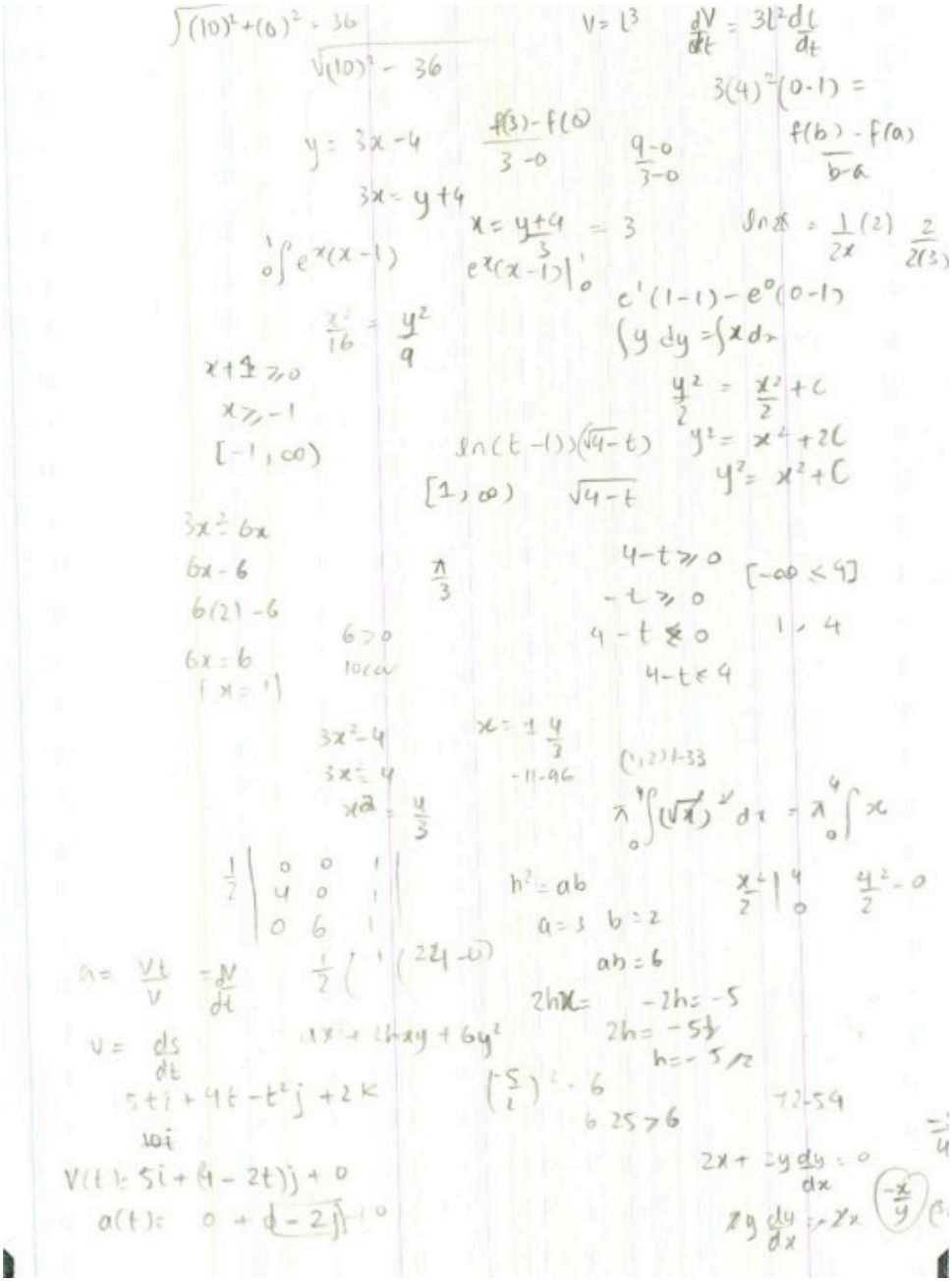








Q. No. 2 Part (I) (Page 1)

$f(x) = \frac{2x-3}{x+1}$

Domain of $f^{-1}(x)$ = Range of $f(x)$

Range of $f^{-1}(x)$ = Domain of $f(x)$

Domain of $f(x) = \mathbb{R} - \{-1\}$ as function becomes undefined.

Now for $f^{-1}(x)$,

let $y = \frac{2x+3}{x+1}$ $\because f(x) = y$

Solving for x

$xy + y = 2x + 3$

$xy - 2x = 3 - y$

$x(y-2) = 3 - y$

$x = \frac{3-y}{y-2}$

$\because x = f^{-1}(y)$

$f^{-1}(y) = \frac{3-y}{y-2}$

Replacing y with x

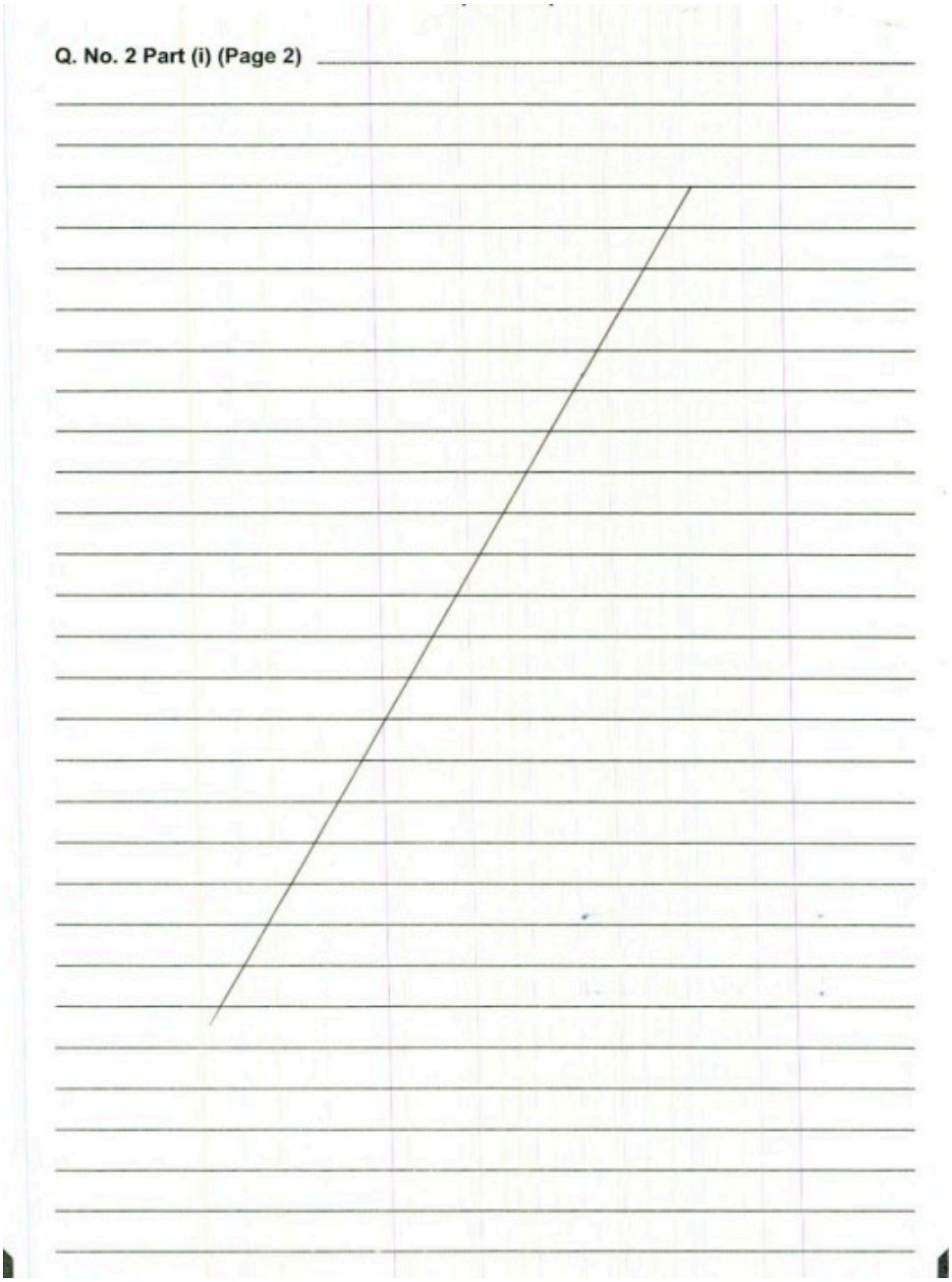
$$\boxed{f^{-1}(x) = \frac{3-x}{x-2}}$$

Now Domain of $f^{-1}(x) = \mathbb{R} - \{2\}$

Therefore, Domain of $f^{-1}(x) = \mathbb{R} - \{2\}$

Range of $f^{-1}(x) = \mathbb{R} - \{-1\}$

Q. No. 2 Part (i) (Page 2)



Q. No. 2 Part (ii) (Page 1)

$y = e^{2x} \ln(x)$

$\frac{dy}{dx} = ?$

$\frac{dy}{dx}\{f(x)g(x)\} = f(x)g'(x) + g(x)f'(x)$

$f(x) = e^{2x} \quad g(x) = \ln x$

Now, differenciating both sides of eq; we get :-

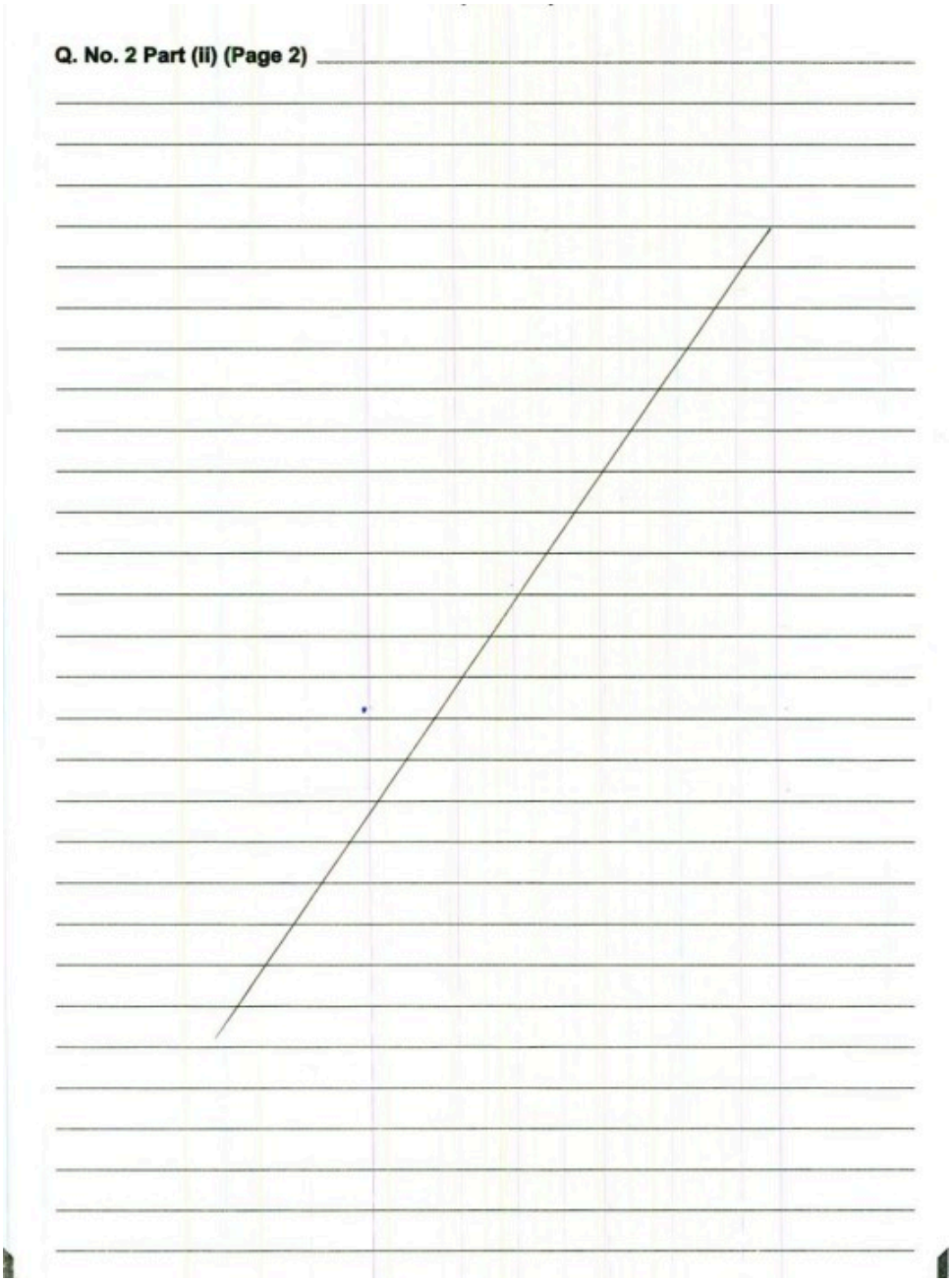
$$\frac{dy}{dx} = (e^{2x})\left(\frac{d}{dx}\ln x\right) + (\ln x)\left(\frac{d}{dx}(e^{2x})\right)$$

$$\frac{dy}{dx} = e^{2x}\left(\frac{1}{x}\right) + \ln x\left(e^{2x}\frac{d}{dx}(2x)\right)$$

$$\frac{dy}{dx} = \frac{e^{2x}}{x} + \ln x\left(e^{2x}(2)\right)$$

$$\boxed{\frac{dy}{dx} = e^{2x}\left[\frac{1}{x} + 2\ln x\right]}$$

Q. No. 2 Part (ii) (Page 2)



Q. No. 2 Part (iii) (Page 1)

$= \int \sin^2 x \, dx$

$\because \sin^2 x = \frac{1 - \cos 2x}{2}$

$= \int \frac{1 - \cos 2x}{2} \, dx$

$= \int \frac{1}{2} dx - \int \frac{\cos 2x}{2} dx$

$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + C$

$= \boxed{\frac{1}{2}x - \frac{\sin 2x}{4} + C}$

$\because \cos 2x = \cos^2 x - \sin^2 x$

$\Rightarrow \cos 2x = (1 - \sin^2 x) - \sin^2 x$

$\Rightarrow \cos 2x = 1 - 2\sin^2 x$

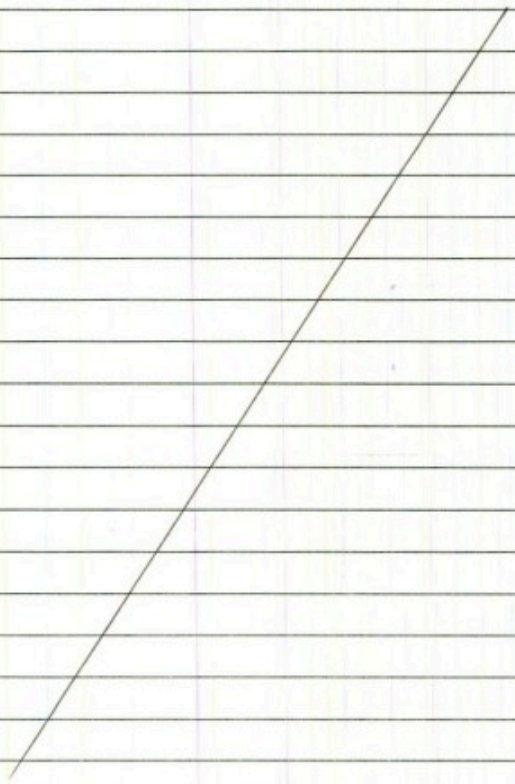
$\Rightarrow \cos 2x - 1 = -2\sin^2 x$

$\Rightarrow \frac{\cos 2x - 1}{-2} = \sin^2 x$

$= \frac{-(1 - \cos 2x)}{2} = \sin^2 x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

Q. No. 2 Part (iii) (Page 2)



Q. No. 2 Part (iv) (Page 1)

$$\frac{dy}{dx} = \frac{y+1}{x-2}$$

Separating variables:

$$\frac{dy}{y+1} = \frac{dx}{x-2} \rightarrow (1)$$

Applying integration on both sides of eq (1)

$$\int \frac{dy}{y+1} = \int \frac{dx}{x-2}$$

$$\ln(y+1) = \ln(x-2) + C$$

$$\ln(y+1) = \ln(x-2) + \ln C \quad \because \ln x + \ln y = \ln xy$$

$$\ln(y+1) = \ln(x-2)(C)$$

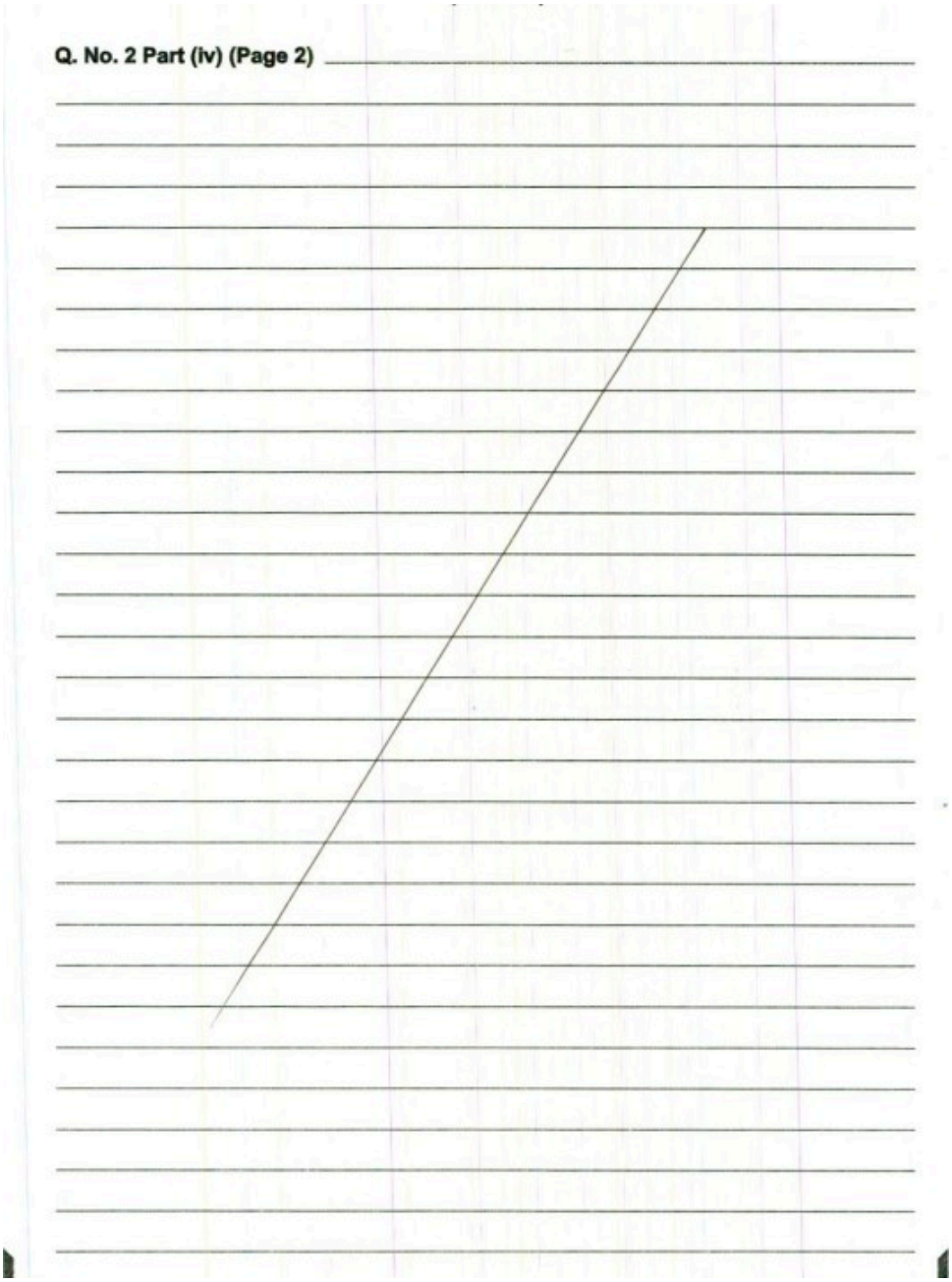
$$e^{\ln(y+1)} = e^{\ln(C(x-2))}$$

$$y+1 = C(x-2) \quad \because e^{\ln} = 1$$

$$y+1 = xC - 2C$$

$$\boxed{y - xC + 2C + 1 = 0}$$

Q. No. 2 Part (iv) (Page 2)



Q. No. 2 Part (v) (Page 1)

$3x^2 - 5xy + 2y^2 = 0$

Coefficient of $x^2 = 3$ & $y^2 = 2$

a) if sum of coefficients of x^2 & $y^2 = 0$ then equations lies on origin

$= a + b$

$= 3 + 2 = 5 \neq 0$ Therefore, line equations do not lie on origin.

b) $3x^2 - 5xy + 2y^2 = 0$

~~$3x^2 - 6xy + xy + 2y^2 = 0$~~

~~$3x(x - 2y) + y$~~

$= 3x^2 - 3xy + 2xy + 2y^2 = 0$

~~$3x(x - y) + 2y$~~

Dividing y^2 on both sides of eq.

$$\frac{3x^2}{y^2} - \frac{5xy}{y^2} + \frac{2y^2}{y^2} = \frac{0}{y^2}$$

$$3\left(\frac{x}{y}\right)^2 - 5\frac{x}{y} + 2 = 0$$

let $\frac{x}{y} = z$ then

$3z^2 - 5z + 2 = 0$

$$z = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$$

$$z = \frac{5 \pm \sqrt{25 - 24}}{6} \Rightarrow z = \frac{5 \pm 1}{6}$$

Q. No. 2 Part (v) (Page 2)

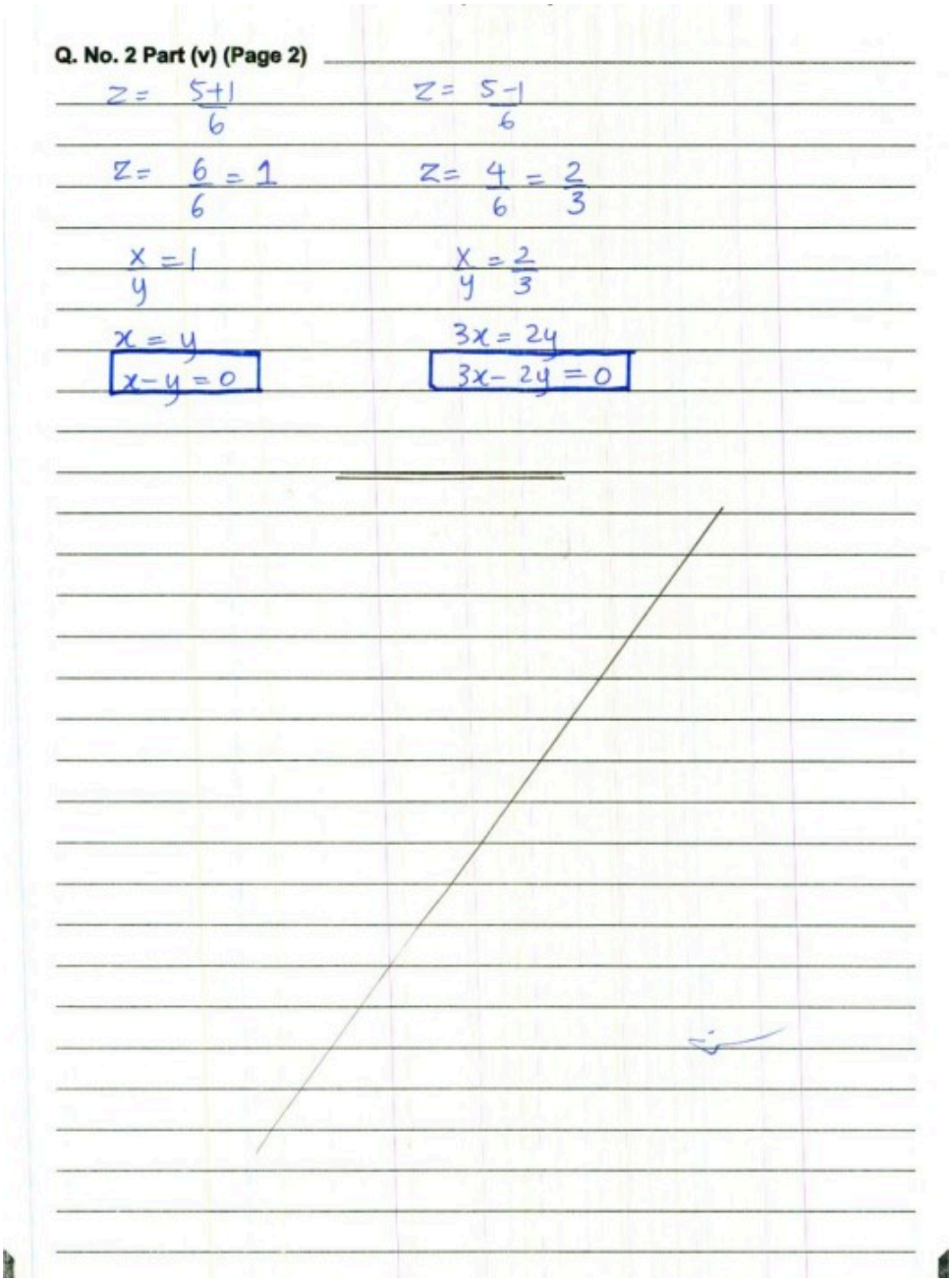
$z = \frac{5+1}{6}$ $\quad z = \frac{5-1}{6}$

$z = \frac{6}{6} = 1$ $\quad z = \frac{4}{6} = \frac{2}{3}$

$\frac{x}{y} = 1$ $\quad \frac{x}{y} = \frac{2}{3}$

$x = y$ $\quad 3x = 2y$

$\boxed{x - y = 0}$ $\quad \boxed{3x - 2y = 0}$



Q. No. 2 Part (vi) (Page 1)

$\gamma(t) = (t^2 - 1)\hat{i} + 3t\hat{j} + \ln(t+1)\hat{k}$

$\gamma'(t) = ?$

$\gamma'(t) = (2t - 0)\hat{i} + 3\hat{j} + \frac{1}{(t+1)}\hat{k}$

$\gamma'(t) = 2t\hat{i} + 3\hat{j} + \frac{1}{(t+1)}\hat{k}$

$\gamma'(2) = 4\hat{i} + 3\hat{j} + \frac{1}{(2+1)}\hat{k}$

$\gamma'(2) = 4\hat{i} + 3\hat{j} + \frac{1}{3}\hat{k}$

Magnitude of $\gamma'(2) = \sqrt{(4)^2 + (3)^2 + \left(\frac{1}{3}\right)^2}$

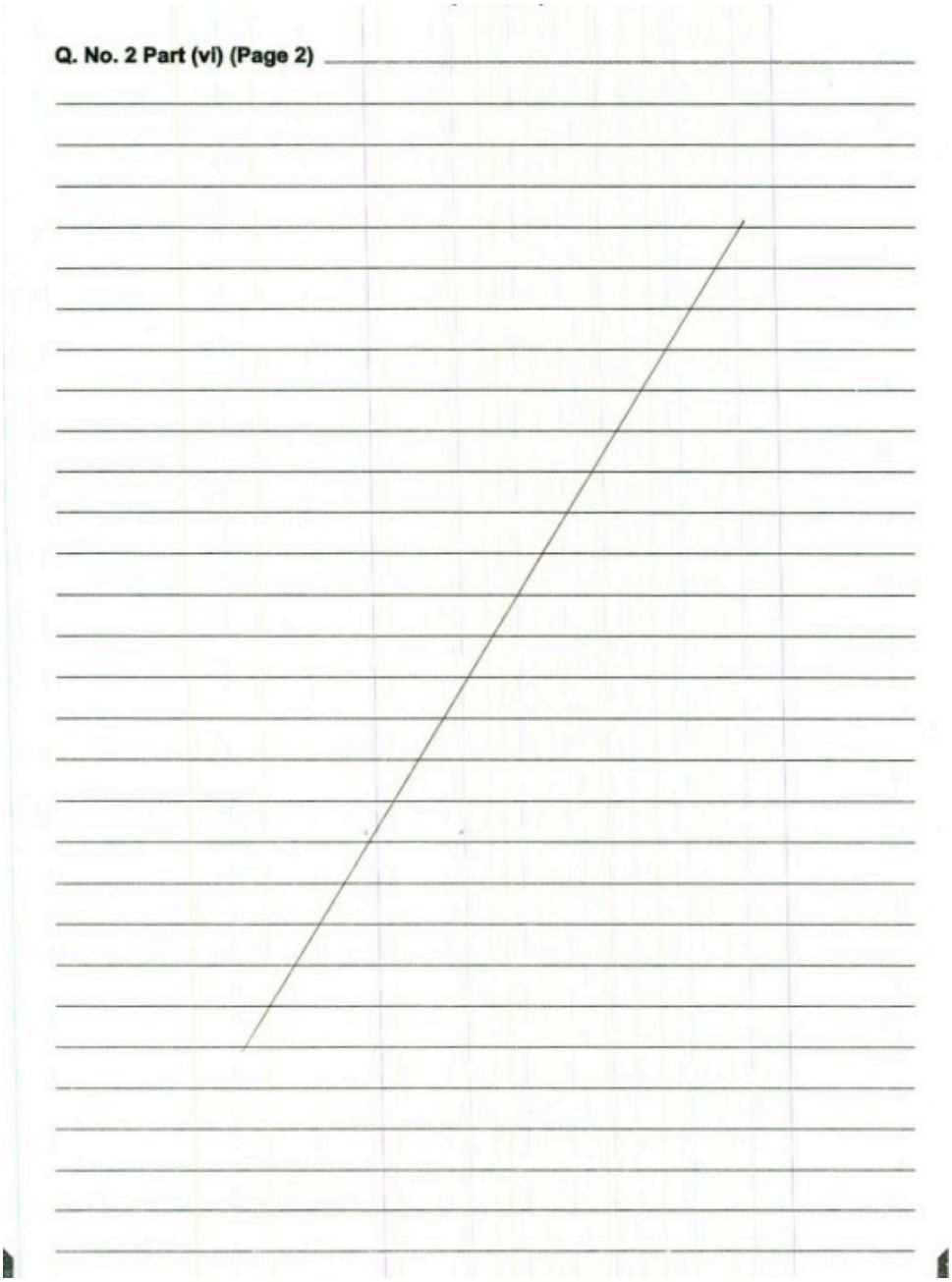
$|\gamma'(2)| = \sqrt{16 + 9 + \frac{1}{9}}$

$= \sqrt{\frac{9(16) + 81 + 1}{9}}$

$= \sqrt{\frac{144 + 81 + 1}{9}}$

$|\gamma'(2)| = \frac{\sqrt{226}}{3} = \boxed{5.011}$

Q. No. 2 Part (vi) (Page 2)



Q. No. 2 Part (vii) (Page 1)

$y^2 = -12x$ origin $(0,0)$

$\because y^2 = -4ax$

Comparing equations

$y^2 = y^2$

$-12x = -4ax$

$a = 3$

a) Vertex = ?

$(y-k)^2 = -4a(x-h)$

Vertex $(h,k) = (0,0)$

b) Focus = ? axis of symmetry : x-axis

focus $(-a,0)$

focus $(-3,0)$

c) Directrix :

$x = a$

$x = 3$

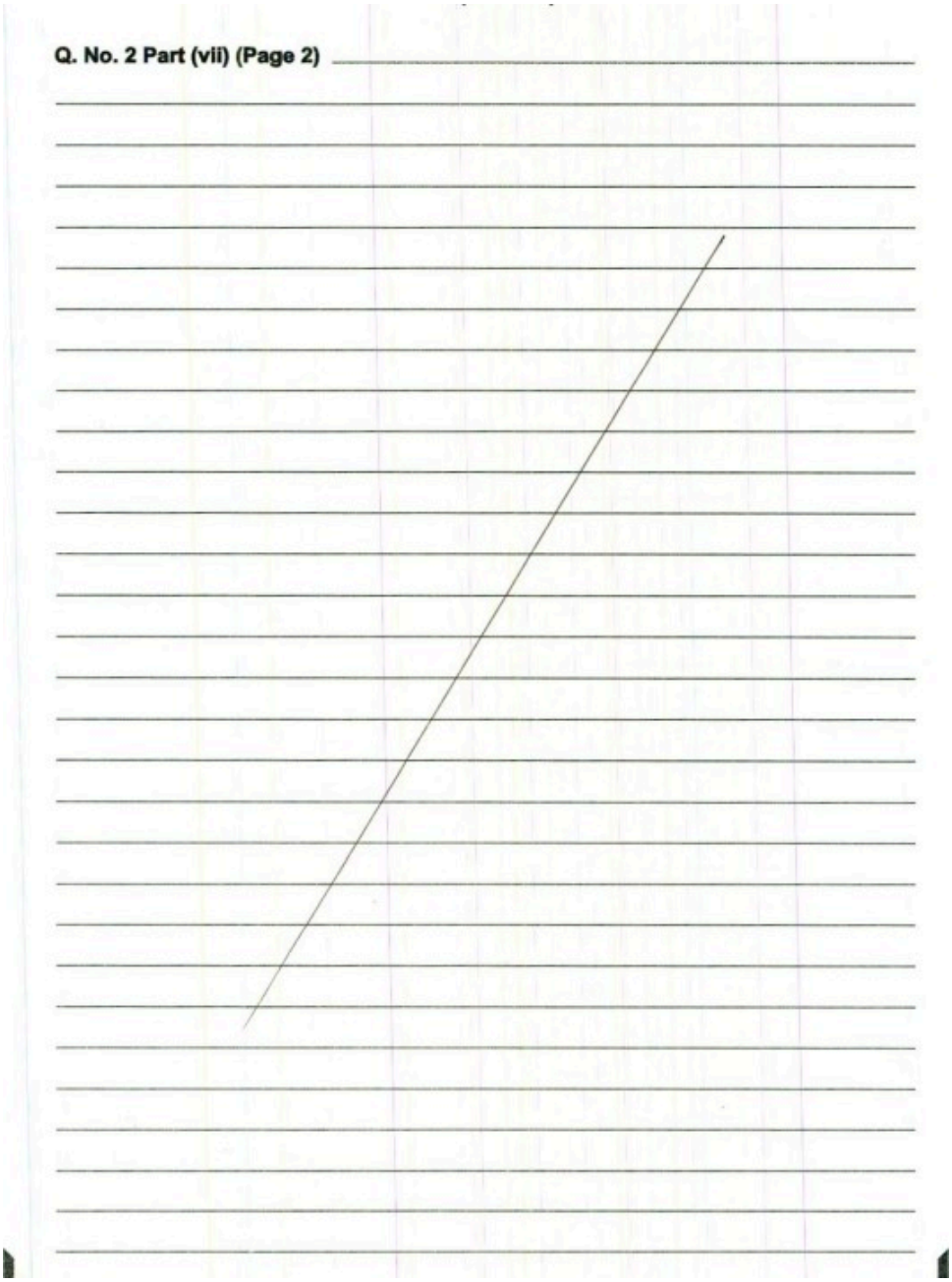
$x - 3 = 0$

d) Length of Latus Rectum

$= |4a|$

$= |4(3)| = 12$

Q. No. 2 Part (vii) (Page 2)



Q. No. 2 Part (viii) (Page 1)

Given:

$r = 10cm$

$\Delta r = dr = 0.05cm$

$\Delta V = dv = ?$ Percentage error in $V = ?$

V of sphere $= \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi r^3$

differentiating both sides of eq

$\frac{dv}{dr} = \frac{4}{3}\pi(3r^2)$

$dv = \frac{4}{3}\pi(3r^2)dr$

$dv = \frac{4}{3}(\pi)(3(10)^2)(0.05)$

$dv = 20\pi = 62.83\ cm^3$

Percentage Error $= \frac{\text{Max Error}}{\text{Original Value}} \times 100\%$

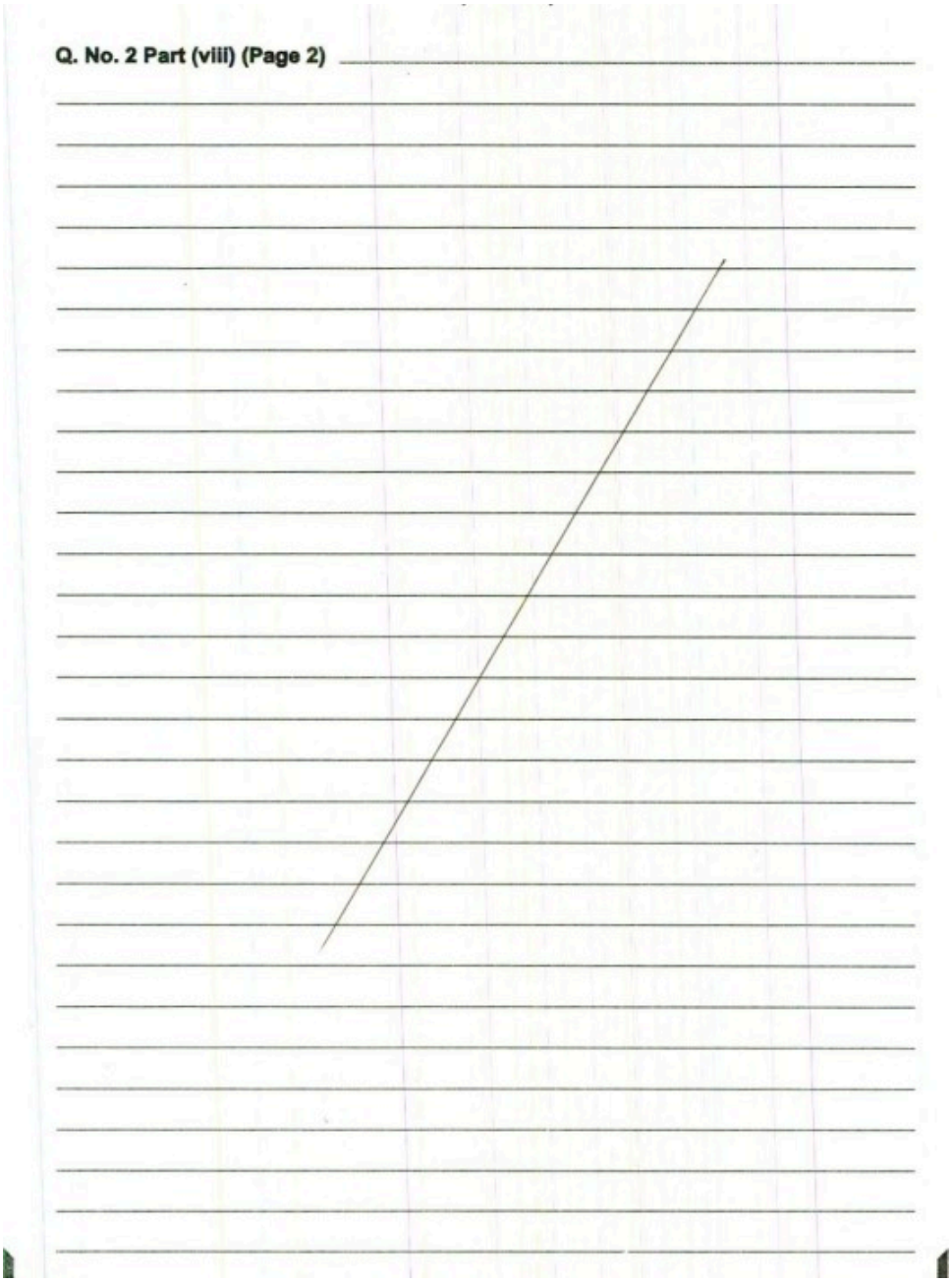
Original Value $= V = \frac{4}{3}\pi(10)^3$

$V = 4188.79\ cm^3$

Percentage Error $= \frac{62.83}{4188.79} \times 100\%$

$= 1.49 \approx \boxed{1.5\%}$

Q. No. 2 Part (viii) (Page 2)



Q. No. 2 Part (ix) (Page 1)

$$\int_0^1 x e^x \, dx$$

$$= \int_a^b f(x)(g(x))\, dx$$

$$\rightarrow \int_a^b = f(x) \int g(x) - \int\int g(x) \frac{d}{dx} f(x)$$

$f(x) = x \Rightarrow g(x) = e^x$

$$= \int_0^1 x e^x dx = \left[x \int e^x - \int\int e^x \frac{d}{dx}(x)\, dx \right]$$

$$= x e^x - \int e^x (1)\, dx$$

$$= x e^x - [1(e^x)] \Big|_0^1$$

$$= x e^x - e^x \Big|_0^1$$

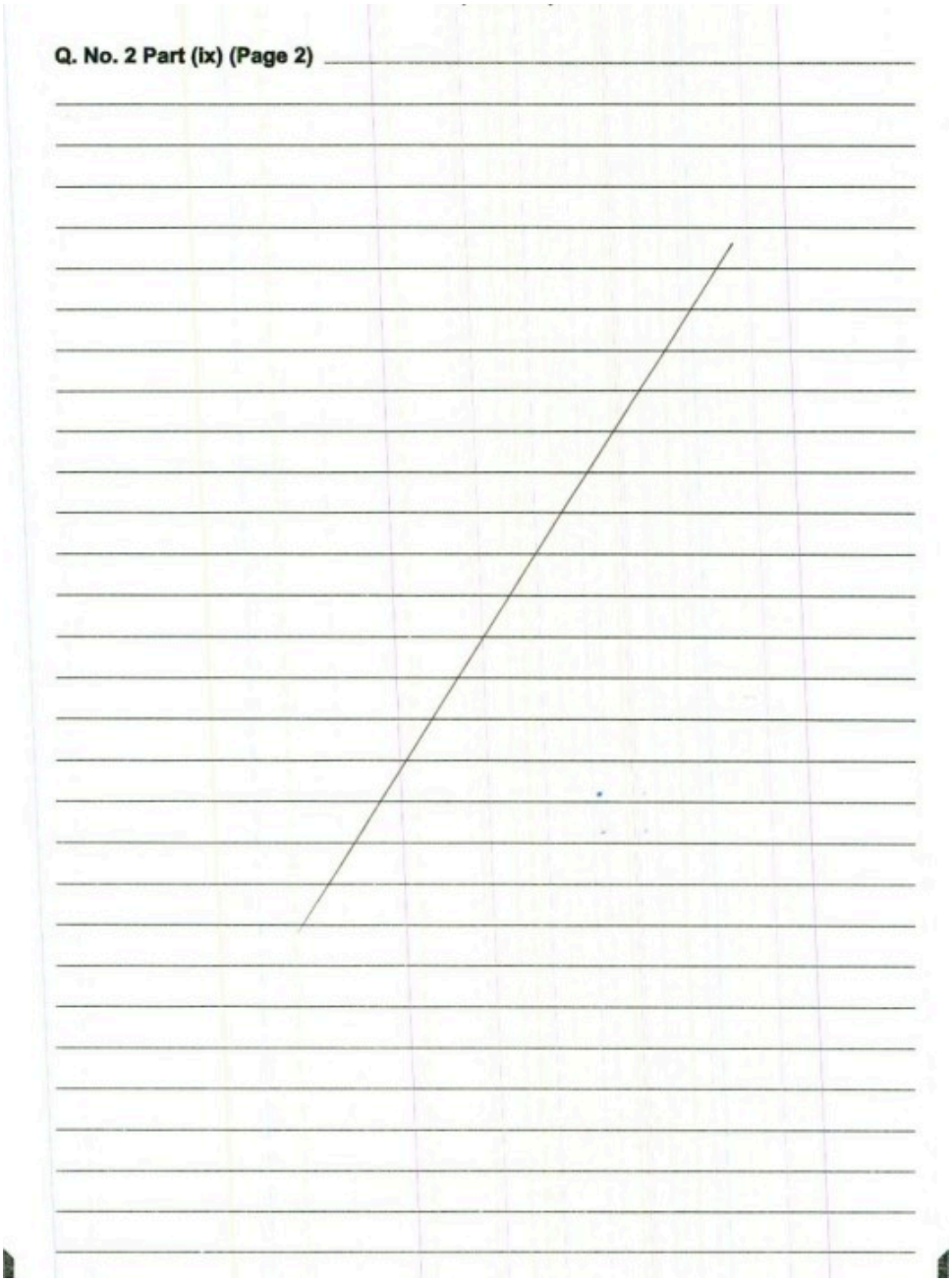
$$= e^x (x - 1) \Big|_0^1$$

$$= e^1(1-1) - e^0(0-1)$$

$$= 0 - e^0(-1)$$

$$= -1(-1) = \boxed{1}$$

Q. No. 2 Part (ix) (Page 2)



Q. No. 2 Part (x) (Page 1)

$y = x^2 + 2$

x-axis equation: $y = 0$

limits $[1, 3]$

~~$A = \int_a^b f(x)\,dx$~~ $\rightarrow$ $A = \int_a^b (\text{upper curve} - \text{lower curve})\,dx$

$x =$	1·5	1·7	2
$y = x^2+2 =$	4.25	4.89	6
$y=0:$	0	0	0

Hence upper curve $= x^2+2$, lower curve $= 0$

$$A = \int_1^3 (x^2+2-0)\,dx$$

$$= \int_1^3 (x^2+2)\,dx$$

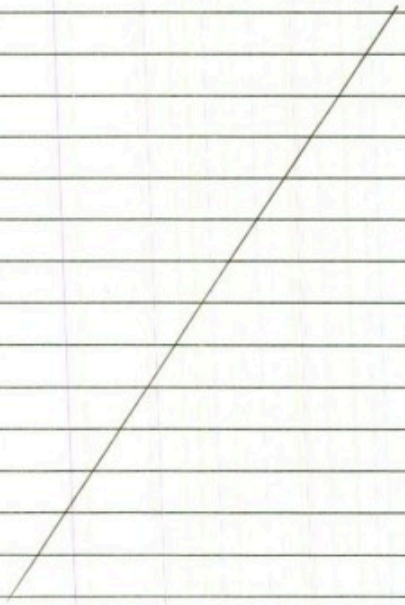
$$= \frac{x^3}{3} + 2x \Big|_1^3$$

$$= \frac{3^3}{3} + 2(3) - \frac{(1)^3}{3} - 2(1)$$

$$= \frac{27}{3} + 6 - \frac{1}{3} - 2$$

$$A = \frac{26}{3} + 4 = \frac{38}{3} = 12.666\ldots$$ 12.67 unit square

Q. No. 2 Part (x) (Page 2)



Q. No. 2 Part (xi) (Page 1)

$$f(x) = \frac{x^2+1}{x^2-1}$$

$f'(x) = ?$

$$\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$f(x) = x^2+1$, $g(x) = x^2-1$

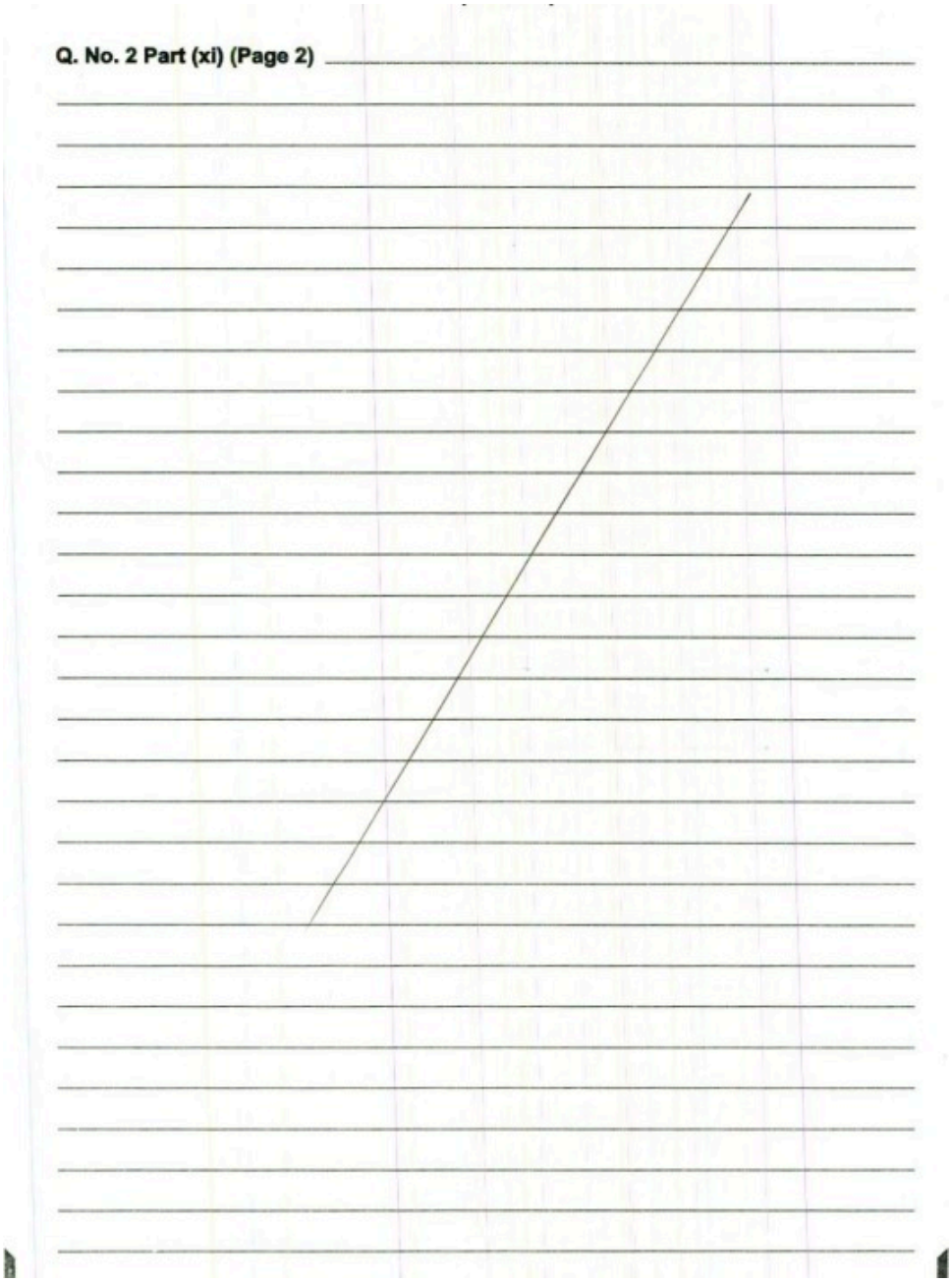
$$\frac{dy}{dx} = \frac{(x^2-1)\left[\frac{d}{dx}(x^2+1)\right] - (x^2+1)\left[\frac{d}{dx}(x^2-1)\right]}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-4x}{(x^2-1)^2}}$$

Q. No. 2 Part (xi) (Page 2)



Q. No. 2 Part (xii) (Page 1)

$f(x) = \sqrt{x-1}$

a) $f^{-1}(x) = ?$

Let $y = f(x)$

$y = \sqrt{x^2-1}$

Taking square on both sides

$y^2 = x-1$

$x = y^2+1$

$x = f^{-1}(y)$

$f^{-1}(y) = y^2+1$

Replace y with x

$\boxed{f^{-1}(x) = x^2+1}$

b) $f(f^{-1}(x)) = x$

L.H.S $= \sqrt{(x^2+1)-1}$

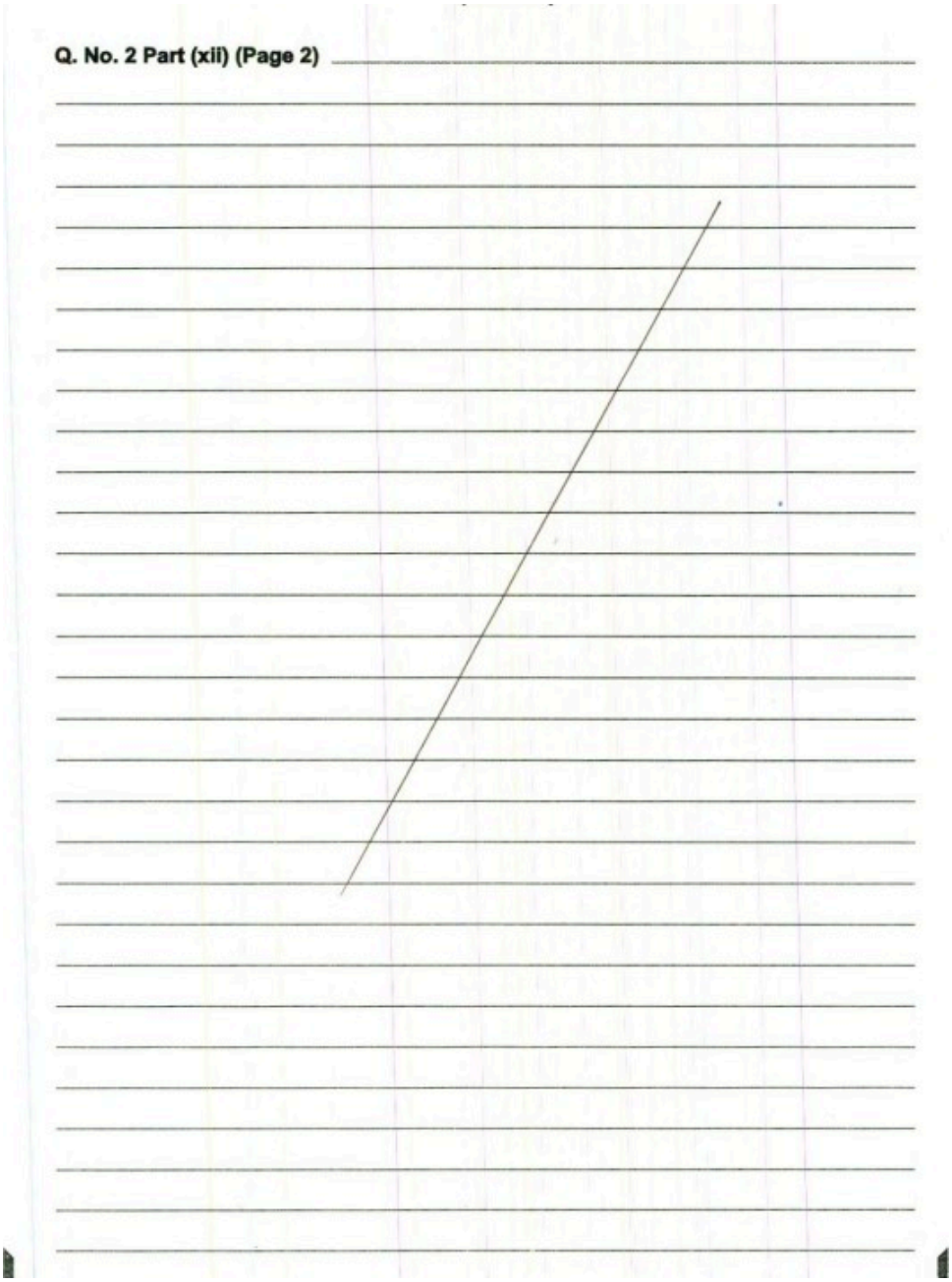
$= \sqrt{x^2+X-X}$

$= \sqrt{x^2}$

$= x$ = R.H.S

Hence proved L.H.S = R.H.S

Q. No. 2 Part (xii) (Page 2)



(Section C)

Q. No. 3 (Page 1)

$$f(x) = \begin{cases} ax^2+bx+2 & ; \quad x \leq 1 \\ 3x+a & ; \quad 1<x<3 \\ \frac{2x^2+b}{x} & ; \quad x \geq 3 \end{cases}$$

function is continuous if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

for $x \to 1$:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} (ax^2+bx+2)$$

$$= a+b+2 \rightarrow \text{eq } ①$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3x+a)$$

$$= 3+a \rightarrow \text{eq } ②$$

$$f(1) = a+b+2$$

according to definition of continuity

$$a+b+2 = 3+a$$

$$b+2 = 3$$

$$b = 3-2$$

$$\boxed{b = 1}$$

~~putting $b=1$ in eq ②~~

$$~~3+a~~ = a+1+2$$

$$= a+3$$

for $x \to 3$:

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} 3x+a$$

$$= 3(3)+a = 9+a \rightarrow \text{eq } ③$$

Q. No. 3 (Page 2)

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{2x^2 + b}{x}$$

$$= \frac{2(3)^2 + b}{3}$$

$$= \frac{18 + b}{3} \rightarrow \text{eq } (4)$$

equating (3) & (4)

$$9 + a = \frac{18 + b}{3}$$

$$27 + 3a = 18 + b$$

$$3a - b = 18 - 27$$

$$3a - b = -9$$

$$3a - b + 9 = 0 \longrightarrow (*)$$

Now put value of $b = 1$ in eq (*)

$$3a - 1 + 9 = 0$$

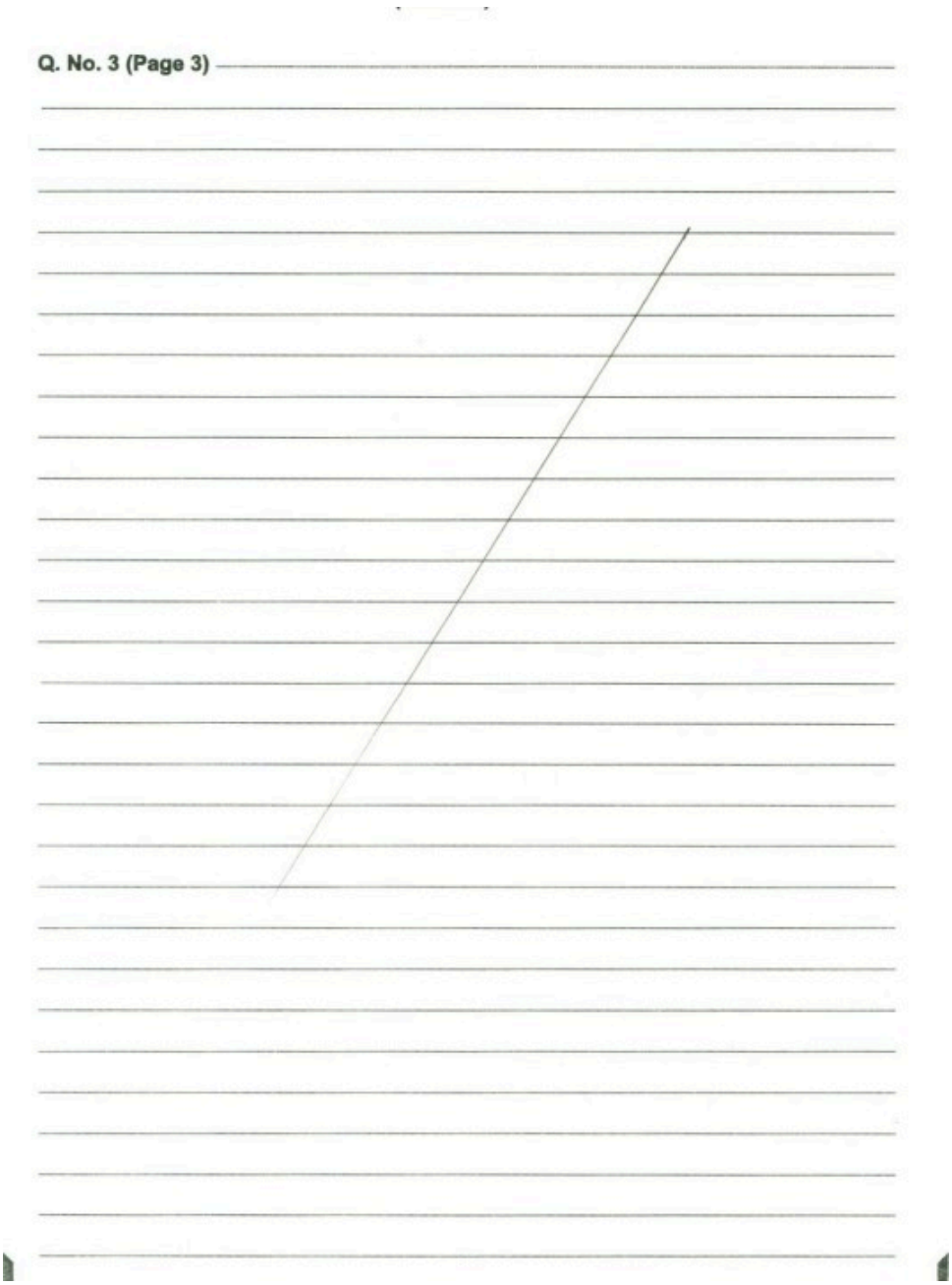
$$3a + 8 = 0$$

$$\boxed{a = \frac{-8}{3}} = -2.667 \approx -2.67$$

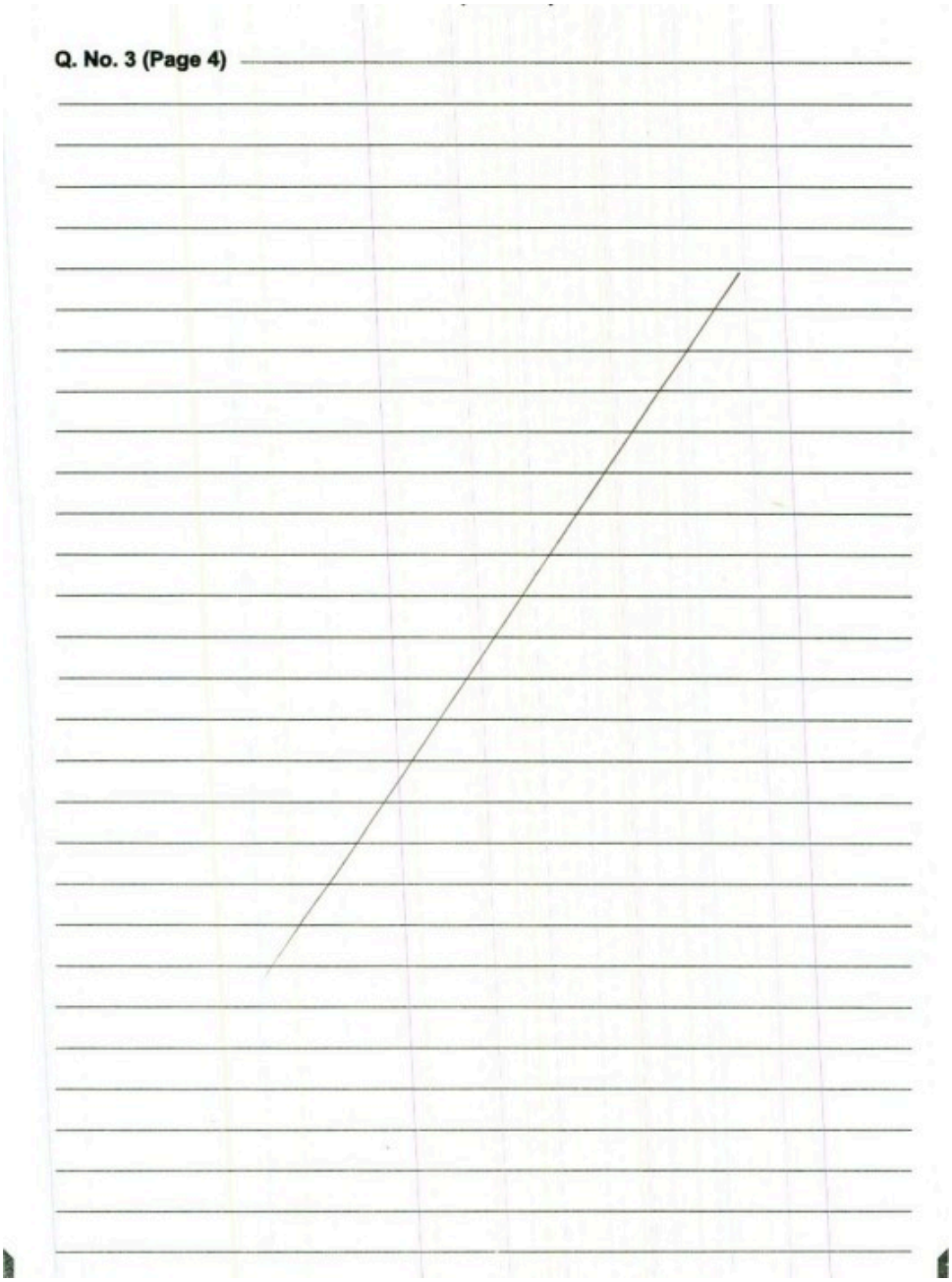
Therefore $\boxed{a = -2.667 \text{ , } b = 1}$

~~value of func at (~~

Q. No. 3 (Page 3)



Q. No. 3 (Page 4)



Q. No. 4 (Page 1)

$f(x) = 2\sin x + \cos 2x, x \in [0, \pi]$

a) Critical points

$f'(x) = 0$

$f'(x) = 2(\cos x) - \sin 2x(2)$

$= 2\cos x - 2\sin 2x$

$2\cos x - 2\sin 2x = 0$

$2\cos x - 2(2\sin x \cos x) = 0$

$\cos x - 2\sin x \cos x = 0$

$\cos x(1 - 2\sin x) = 0$

$\cos x = 0$

$x = \cos^{-1}(0)$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \ldots (2n+1)\frac{\pi}{2}$

$x = \frac{\pi}{2} \in [0, \pi]$

$1 - 2\sin x = 0$

$1 = 2\sin x$

$\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6}$

$\sin(+)$ in I & II Q

In II Q, $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\} \in [0, \pi]$

Critical points within $x = [0, \pi]$ are

$= \boxed{\left\{\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}}$

b) Local maxima or minima

Q. No. 4 (Page 2)

$f''(x) = 2(-\sin x) - 2(\cos 2x)(2)$

$f''(x) = -2\sin x - 4\cos 2x$

for $x = \pi/2$

$f''\left(\frac{\pi}{2}\right) = -2\sin\frac{\pi}{2} - 4\cos 2\left(\frac{\pi}{2}\right)$

$= -2\sin\frac{\pi}{2} - 4\cos\pi$

$f''\left(\frac{\pi}{2}\right) = 2 > 0$, Therefore at $x = \frac{\pi}{2}$, relative minima exists

for $x = \frac{\pi}{6}$

$f''\left(\frac{\pi}{6}\right) = -2\sin\frac{\pi}{6} - 4\cos 2\left(\frac{\pi}{6}\right)$

$= -3 < 0$, so at $x = \frac{\pi}{6}$, relative or local maxima exists

for $x = 5\pi/6$

$f''\left(\frac{5\pi}{6}\right) = -2\sin\left(\frac{5\pi}{6}\right) - 4\cos 2\left(\frac{5\pi}{6}\right)$

$= -3 < 0$ relative ~~minima~~ maxima exists

Relative maxima at $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

Relative minima at $x = \frac{\pi}{2}$

c) if $f''(c) > 0$ absolute maxima exists and for $f''(c) < 0$ absolute minima exists

Q. No. 4 (Page 3)

$f'(x) = 2\cos x - 2\sin 2x$

at $x = \frac{\pi}{2}$

$f'(\frac{\pi}{2}) = -2 < 0$ absolute minima

at $x = \frac{\pi}{6}$

$f'(\frac{\pi}{6}) = 0 \rightarrow$ no maxima or minima

at $x = \frac{5\pi}{6}$ $f'(\frac{5\pi}{6}) = 0 \rightarrow$ No maxima or minima

at $x = 0$

$f'(0) = 2\cos 0 - 2\sin 2(0)$

$f'(0) = 2 > 0$ absolute maxima

$f(x) = 2\sin x + \cos 2x$

at $x = \frac{\pi}{2}$

$f(\frac{\pi}{2}) = 1.5 > 0$ maxima

at $x = \frac{\pi}{6}$

$f(\frac{\pi}{6}) = \frac{3}{2} > 0$ maxima

at $x = \frac{5\pi}{6}, f(\frac{5\pi}{6}) > 0$

$f(\frac{5\pi}{6}) = \frac{3}{2} > 0$ maxima

at $x = 0$

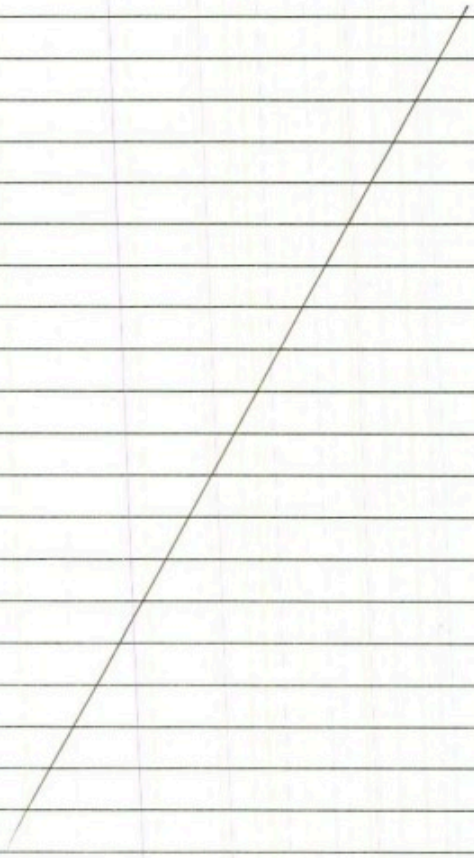
$f(0) = 1 > 0$ maxima

at $x = \pi$

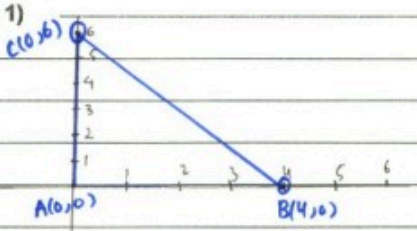
$f(\pi) = 1 > 0$ maxima

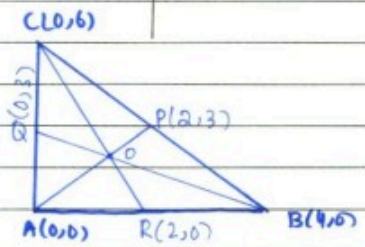
maximum value of function = 1.5 at $x = \{\frac{\pi}{6}, \frac{5\pi}{6}\}$

Q. No. 4 (Page 4)



Q. No. 5 (Page 1)





P is midpoint of BC $= \left(\frac{4+0}{2}, \frac{6+0}{2}\right)$

$\boxed{P = (2,3)}$

Q midpoint of AC $= \left(\frac{0+0}{2}, \frac{6+0}{2}\right)$

$\boxed{Q = (0,3)}$

R midpoint of AB $= \left(\frac{0+4}{2}, \frac{0+0}{2}\right)$

$\boxed{R = (2,0)}$

AP, BQ & CR must be concurrent:

$\because$ eq of line $= y - y_1 = m(x - x_1)$

Slope of AP $= \frac{3-0}{2-0} = \frac{3}{2}$

Eq of AP $\Rightarrow y - 3 = \frac{3}{2}(x-2)$

$2y - 6 = 3x - 6$

Q. No. 5 (Page 2)

$\boxed{3x-2y=0}$ — ①

Slope of BQ = $\frac{3-0}{0-4} = \frac{3}{-4} = \frac{-3}{4}$

Eq of BQ = $y-3 = \frac{-3}{4}(x-0)$

$4(y-3) = -3x$

$4y-12+3x=0$

$\boxed{3x+4y-12=0}$ → ②

Slope of CR: $\frac{0-6}{2-0} = \frac{-6}{2} = -3$

Eq of CR : $y-0 = -3(x-2)$

$y = -3x+6$

$3x+y=6$

$\boxed{3x+y-6=0}$ → ③

For three lines to be concurrent, the determinent must be equal to zero.
OR the point must satisfy three equations

Solving eq ① & ②

$3x-2y=0$

$-3x+4y=-12$

$-6y=-12$

$y=2$

put $y=2$ in eq ①

Q. No. 5 (Page 3)

$3x - 2(2) = 0$

$3x - 4 = 0$

$3x = 4$

$x = \frac{4}{3} = 1.33$

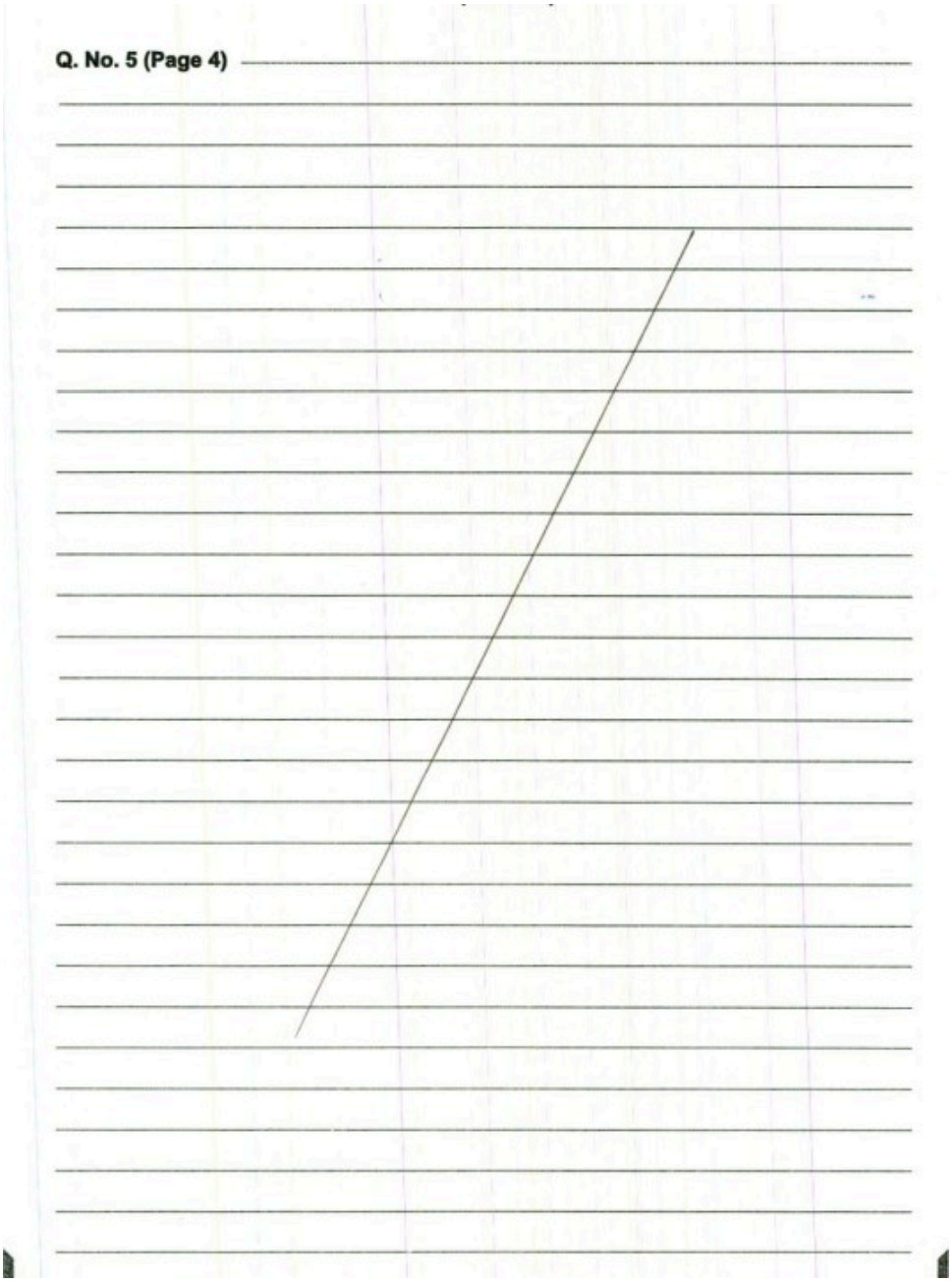
point of concurrency $(1.33, 2)$

put point in eq (3)

$3(1.33) + 2 - 6 \approx 0.01 \neq 0$

So lines are concurrent at point (1.33, 2) called centroid.

Q. No. 5 (Page 4)



Q. No. 6 (Page 1)

$$\frac{dy}{dx} = \frac{x^2+y^2}{xy}$$

Let $y = ux$ where u is an independent variable then

$$\frac{dy}{dx} = u\frac{dx}{dx} + x\frac{du}{dx}$$

$$\frac{dy}{dx} = u(1) + x\frac{du}{dx}$$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

putting values of y & $\frac{dy}{dx}$ in eq

$$u + x\frac{du}{dx} = \frac{x^2+(ux)^2}{x(ux)}$$

$$u + x\frac{du}{dx} = \frac{x^2+u^2x^2}{x^2u}$$

$$u + x\frac{du}{dx} = \frac{x^2(1+u^2)}{x^2u}$$

$$x\frac{du}{dx} = \frac{1+u^2}{u} - u$$

$$x\frac{du}{dx} = \frac{1+u^2-u^2}{u}$$

$$x\frac{du}{dx} = \frac{1}{u}$$

$$u\,du = \frac{dx}{x}$$

Taking integral on both sides

Q. No. 6 (Page 2)

$$\int u\,du = \int \frac{dx}{x}$$

$$\frac{u^2}{2} = \ln x + C$$

∵ putting $u = \frac{y}{x}$

$$\frac{(y/x)^2}{2} = \ln x + C$$

$$\frac{y^2}{2x^2} = \ln x + C$$

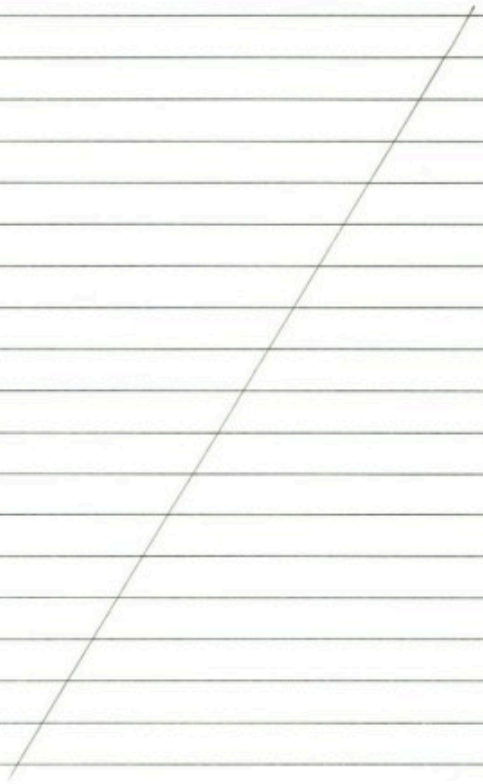
Multiplying whole eq with 2

$$2\left(\frac{y^2}{2x^2}\right) = 2\ln x + 2C$$

$$\boxed{\frac{y^2}{x^2} = 2\ln x + C}$$

This is called general ~~equation~~ solution of homogenous equation

Q. No. 6 (Page 3)



Q. No. 6 (Page 4)

